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METHODOLOGY OF PSYCHOMETRIC STUDIES IN PSYCHOACOUSTICS

Metodologia badań psychometrycznych w psychoakustyce

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ABSTRACT:

A general philosophical approach to methodology of psychoacoustics is provided. The idea of sound classes and psychometric functions is described. The role of modern conceptions such as: rough sets and fuzzy sets in the analysis and interpretation of psychoacoustical data is pointed out. Illustrating examples are given.

1. INTRODUCTION

Psychoacoustics is an interdisciplinary empirical science with methodology adopted in part from psychology and in part from physics. Methods and specific techniques of psychoacoustical studies are now well documented in scientific literature [1] It is usually desirable to describe psychoacoustical data not only qualitatively but also quantitatively in order to describe psychological phenomena in terms of mathematical formulas. Extracting and mathematical analysis of numerical data obtained from psychological (and psychoacoustical) experiments is the main tasks of *psychometry*. Due to uncertainty and complexity of human reactions to various stimuli, methodology of psychometric studies is not always obvious so that many different methods of data extraction, analysis and interpretation are now applied. In particular, psychometric functions are commonly used to describe mathematically psychoacoustical effects caused by various sound stimuli. There is a problem of determination of a psychometric function suitable for a given research problem. The classical statistical approach to psychometric studies is neither always sufficient nor necessary to analyse and interpret properly psychoacoustiocal data.

The aim of this work is to provide theoretical foundations of contemporary psychometry in psychoacoustics based on modern mathematical conceptions such as: *sound* *classes*, *rough sets*, *fuzzy sets* and *data mining*. Using these ideas, general definitions and methods of determination of psychometric functions are described.

2. SOUNDS AND CLASSES

The human auditory system is subjected to variety of sounds that are perceived and interpreted. A given sound may be perceived by a listener as to be *adverse* or *pleasant*, *silent* or *loud*, *harmonic*, *dissonance impulsive*, *rough*, *sharp*, *modulated* etc. Therefore, it can be distinguished the set S of all possible auditory sensations (subjective features) as determined states of the human brain. Since the set S can be treated as the collection of adjectives naturally associated with audible sounds, it is obviously finite.

In the other side, any audible sound can be approximately described by a finite set of attributes as quantitative (real) values representing its physical properties [1]. There are classes (categories) consisting sounds of similar physical properties and distinguished from other classes by categorical (qualitative) attributes. A class X containing sounds described by the attributes $x_1,..., x_n$ can be interpreted as the space of points $\mathbf{x} = (x_1 ,..., x_n) \in R^n$. According to the general geometric philosophical approach to psychoacoustics [2], any useful class X is an uncountable simply-connected and compact topological manifold when the attributes $x_1,..., x_n$, uniquely identifying sounds belonging to the class, are treated as coordinates on X. The number of attributes n and their choice determines the accuracy and aims of the model of sounds.

3. PSYCHOACOUSTICAL MAPS AND CONCEPTS ON SOUND CLASSES

In psychoacoustical experiments usually sensations induced by sounds belonging to a certain class X are studied. For any subjective feature $s \in S$ the relation between sounds $\mathbf{x} \in X$ and evaluated strength of the corresponding sensations can be represented by a *psychoacoustical map* $\pi_S : X \rightarrow Q \subset R$. The map, determining discrete *scaling* of psychological sensations, is identified by subjects. Human brain has certain ability to judge the magnitude of the perceived feature of sounds with respect to a given scale. This ability is not quite natural but can be improved by training. Any subject can relatively easy judge whether a feature $s \in S$ is (or is not) noticed in a sound. This is the simplest case in which decision process can be described by a binary map $\pi_S : X \rightarrow \{0, 1\}=Q$, where the values 0, 1 correspond to the presence and absence of the feature, respectively. However, it is also possible to ask a subject to estimate the magnitude of a given sound feature more precisely by using a k-level scale. In such a case a function $\pi_S : X \rightarrow \{0, 1, 2,..., k\}= Q$, $k >2$ has to be assumed in order to improve quantitative evaluation of the presented sounds.

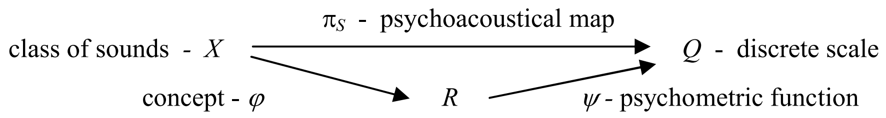
In the other side, looking for relations between physical properties of sounds and the corresponding psychological sensations in human, it is logical to consider continuous real functions $\varphi : X \rightarrow R$ (*concepts* on the space X) representing physical quantities associated with sounds and describing the sounds globally (i.e. independently on the particular choice of the attributes $\mathbf{x} = (x_1,..., x_n)$ as co-ordinates on the space X).

4. PSYCHOMETRIC FUNCTIONS IN PSYCHOACOUSTICS

The notion of *psychometric function* in psychoacoustics means a map assigning subjective evaluations of auditory sensations caused by certain sounds to real numbers describing their physical properties. Psychometric functions establishe certain relations between physical properties of sounds and their psychological features. Therefore, the knowledge of a psychometric function enables to express psychological facts quantitatively as exact lows.

Using the described formalism of psychoacoustical maps and concepts on sound classes one can easily define psychometric functions. To do this let us consider the following

Definition : A concept $\varphi : X \to R$ and a psychoacoustical map $\pi_S: X \to Q$ are said to be in accordance if there is a function $\psi : R \to Q$ such that the following diagram



is commutative i.e. $\psi(\varphi(\mathbf{x})) = \pi_S(\mathbf{x})$ for any $\mathbf{x} \in X$.

If this is the case then the function ψ is called a *psychometric function* associated with the feature s and will be denoted by ψ_S. Similarly, the concept φ, describing physical causes of the subjective feature s will be denoted by φ_S.

5. ROUGHNESS OF PSYCHOMETRIC FUNCTIONS AND SCALES

The above general definition of psychometric functions does not taken into account unavoidable uncertainty of human judgements. Namely, as it follows from psychological tests, independently on the studied sound feature $s \in S$, there may be sounds always evaluated as having the assumed feature s on the unique level $j \in Q = \{0, 1, 2,..., k\}$ as well as sounds not classified uniquely. This means that psychological functions are multi-valued functions in principle. Even judgements regarding the repeated sound, provided by the same subject, need not be coinciding. Hence, assuming that the considered psychological processes are stationary, a psychological function π_S can be determined by the set of values Q and probabilities $p_{s,j}(\mathbf{x})$ that a sound $\mathbf{x} \in X$ presented to a subject will be evaluated at the level $j \in Q$.

In consequence, any discrete psychological scale must be rough (fuzzy). This can be explained by a natural noise disturbing bio-electric processes. There is also a purely topological explanation of scale roughness. Indeed, suppose that a psychoacoustical map π_S: $X \to Q$ is a uni-valued function. It is logical to require that sounds similar to a given sound $\mathbf{x}_0 \in X$ (i.e. sounds belonging to a certain neighbourhood $U(\mathbf{x}_0) \subset X$ of $\mathbf{x}_0$) will be classified on the same level. This requirements means that $\pi_S^{-1}(j)$, $j = 0, 1,..., k$ should be open set in the space X i.e. the psychological map π_S should be a continuous function if $Q=\{0, 1, 2,..., k\}$ is treated as a topological space with the discrete topology. Hence, the sets $\pi_S^{-1}(j)$, $j = 0,1,..., k$ should be disjoint and should cover the space X. However, this is impossible because X is a simply-connected space. Hence, there must exists sounds not uniquely classified i.e. π_S must be a multivalued map Furthermore, due to compactness of the space X, any countable or continuous scale can be practically reduced to a finite scale.

Let the notion $\pi_S(\mathbf{x}) = j$ ($\pi_S(\mathbf{x}) \equiv j$) means that the sound $\mathbf{x}$ can be (is always) evaluated on the level $j \in Q$. Then, the set of all sounds classified as to has the magnitude of the feature s on the level $j \in Q$ can be denoted as $X_{s,j} = \{\mathbf{x} \in X : \pi_S(\mathbf{x}) = j \in Q\}$. Applying the above notations it is easy to notice that the map π_S for any sound feature $s \in S$ determines a frame rough structure on the space X given by the covering composed of open sets $X_{s,j} = \{\mathbf{x} \in X : \pi_S(\mathbf{x}) = j\}, j \in \{0, 1, 2,..., k\} = Q$. The sets $X_{s,j}$, $j = 0, 1,..., k$ are obviously *rough sets* with their lower approximations $Y_{s,j} = \{\mathbf{x} \in X : \pi_S(\mathbf{x}) \equiv j\}$ and rough boundaries $Z_{s,j} = =X_{s,j} - Y_{s,j}$, $j = 0, 1,..., k$, respectively. The frame rough structure $\{X_{s,j}, Y_{s,j}, Z_{s,j} : j = 0, 1,..., k\}$ on the space X together with the classification probabilities $p_j(\mathbf{x})$, $j = 0,1,...,k$, $\mathbf{x} \in X$ constitute mathematical representation of the pschoacoustical phenomenon associated with the sound class X and the particular feature $s \in S$. The two main problems arise here: how to determine the rough structure and the probabilities and how to interpret such information in terms of comprehensive psychological statements?

6. METHODOLOGY OF PSYCHOMETRIC STUDIES

The main methodological problem of psychometry concerns determination suitable concepts, scales psychoacoustical maps and psychometric functions on the assumed class of sounds. The basic stages of psychometric studies are described here below in details.

Once a sound feature $s \in S$ and the space of sounds X are chosen, a psychological scale Q of a suitable number of levels k has to be assumed. A small number of levels facilitates subjective judgement although may not be able to ensure sufficient description accuracy of the studied sound properties. In the other side, a too large number of levels improves description resolution at the expense of increased judgement uncertainty. The scale levels can be either individually subjectively interpreted by subjects or explained by operational definitions before the experiments. In the latter case subjects' judgements are usually more precise and repeatable. Hence, it is valuable if each level would has a clear qualitative explanation (operational definition). In the case of individual absolute judgement the number of scale levels can be bounded by the criterion that only rough boundaries associated with neighbouring scale levels intersect each other i.e.

$X_{s,j} \cap X_{s,j+1} \neq \phi$ for $j = 0,1,...,k$-1 and $X_{s,i} \cap X_{s,j} = \phi$ in other cases for which $i \neq j$.

At this stage, when the space X of sounds, the studied sound feature $s \in S$ and the psychological scale Q is fixed a finite group of subjects G should be prepared for the experiments. A typical psychoacoustical experiment consists in presentation a sequence of sounds $\mathbf{x}^{(1)}, ..., \mathbf{x}^{(m)} \in X$ to each subject $g \in G$. The subjects are asked to answer a question about the presence (absence) of the feature $s \in S$ and its magnitude in the presented sound Each presentation of a sound $\mathbf{x} \in X$ to a subject $g \in G$ provides information in the form of a record $(g, \mathbf{x}, j)_s$ where $j \in Q$ is a result of the subject's judgement determining the psychoacoustical map π_S. Thus, the existence of the map $\pi_S$on the assumed class of sounds X and for any $s \in S$ is empirically justified although it cannot be determined exactly. This is because psychoacoustical maps are usually defined on a continuous space X while the number of numerical records available from experiments is always finite.

The methodological question arise here about an optimal choice and order of the presented sounds that ensure the best discrete representation of the continuous rough structure on the space X. The selection of sounds for presentation may be either adaptive (dependent on the previous outcomes from the experiment), random or fixed i.e. chosen in advance before testing. In the later case we say that the method of *constant stimuli* is applied.

At this stage the following order of adaptive psychoacoustical studies can be recommended. Firstly, a finite set of sounds uniformly dispersed on the space X can be presented to a group of subjects G in order to obtain preliminary, approximate information on the frame rough structure and particularly on rough boundaries. Since the space X is compact, such a set always exists. Then, once the frame rough structure on X is determined, it is logical to study more precisely what is going in the regions of rough boundaries. At this stage at least two ways of further studies are possible - one, implementing conceptions of rough sets and another, based on the fuzzy sets theory.

In order to apply rough sets theory it is necessary to discretise all the attributes (x_1,..., x_n). In this way the whole space X is divided into small boxes. The sounds represented by points belonging to a unique box are assumed to be psychologically equivalent in this approach. Repetitive presentation of a number of sounds chosen from boxes intersecting with rough boundaries enables to estimate classification probabilities for each box. On this basis the so-called *rough membership functions* can be determined. Then, by using the standard procedures in the rough set theory, the laws of membership of sounds to the rough subclasses and/or rough boundaries $X_{s,\ j}$, $Y_{s,\ j}$, $Z_{s,\ j}$, respectively, can be easily expressed in terms of certain *IF-THEN* rules depending on the sound attributes (x_1,..., x_k). It can be also deduced which of the attributes (i.e. physical properties) are significant for perception of the assumed sound feature s.

In the fuzzy set approach the so-called *fuzzy membership functions* are identified on the rough boundaries in the space X with continuous (non-discretised) attributes. In order to do this a number of sounds chosen from the rough boundaries should be presented to the subjects and a particular (e.g. trapezoid or piece-wise linear), parameterised form of the membership functions has to be assumed. Then, an approximate parameter identification of the membership functions can be performed on the basis of relatively small number of numerical. records. It is also possible to perform a non-parametric (statistical) identification of the fuzzy membership function e.g. by applying the *yes-no method* and a large number of repetitive presentations of sounds belonging to a rough boundary.

In practice, usually only the local rough structure between neighbouring regions $X_{s,j}$, $X_{s,j+1}$ on the sound space X is studied by changing a selected attribute in the subsequent presented sounds. In general, such a procedure (the *series method*) can be interpreted geometrically as a sequence of points $\mathbf{x}^{(1)}$, ..., $\mathbf{x}^{(m)}$ along a continuous curve $\mathbf{x}(t)$, $t \in [t_0, t_m\ ]$ running from the region $X_{s,\ j}$ to $X_{s,\ j+1}$ across their rough boundaries. Since there is a one-to-one correspondence between the curve parameter t and $\mathbf{x}(t)$, there is a local concept φ such that $t = \varphi(\mathbf{x})$ on the curve. Hence, it is possible to identify a fuzzy membership function $\mu\ (\varphi)$ of the set $X_{s,j}$ on the basis of empirical records $(\varphi(\mathbf{x}^{(i)})=t_i,\ \pi_z(\mathbf{x}^{(i)}))$, $i = 1,\ldots,m$. In the common case, the concept φ coincides with a selected attribute and the corresponding membership function is determined for fixed the remaining attributes.

Descending and ascending series of sound stimuli are usually applied on the method of series. Both the initial sound $\mathbf{x}^{(1)}$ and the step Δt are essential. At the beginning a lager step is recommended in order to approximate recognition of the rough boundary. The quantities $\mathbf{x}^{(1)}$ and Δt are chosen properly if the change of the subjective level from j to j+1 and v.v. is perceived always within the same section (t_i , t_{i+1}). At the final stage a smaller step Δt and the method of constant stimuli with sounds belonging to the rough boundary can be recommended.

7. EXEMPLARY PSYCHOACOUSTICAL IMPLICATIONS

The hearing threshold for a class of tones as a just noticeable level is a classical example of a notion that cannot be properly defined without the idea of rough (fuzzy) sets and a fuzzy membership function. Similarly, the perception threshold of any sound feature or component is a fuzzy notion in principle.

The physical absolute level of a noise or its mean square acoustical pressure are examples of physical concepts on a class of broad-band sounds. The relation between the physical level of a sound and its perceived loudness is a classical example of a simple psychometric function [1].

Echo perception can be easily studied on the three dimensional space of stimuli. The average level of the direct signal, the relative level of the delayed signal and the delay constitute a convenient set of attributes. The rough structure on the space of stimuli and fuzzy membership functions dependent on the delay can be identified in laboratory experiments.

Sound sharpness can be studied on the space of pairs of audible tones. The tones amplitudes and frequencies can be taken as useful attributes of the presented stimuli.

In order to study perception of the incidence angle of a sounds emitted by a concentrated source, the circle around the subject head has to be divided into several fuzzy sectors reflecting perception resolution of the human auditory system. This is the one-dimensional problem with the number of scale levels coinciding with the number of angle sectors. The corresponding fuzzy trapezoid membership functions dependent on the incident angle can be identified in the anechoic room.

There are more complex psychoacoustical maps estimated empirically (for example the speech intelligibility score in a classroom) but the associated physical concept is not known as yet.

It is worth to notice that psychometric functions considered in this work describe only those sound properties that can be expressed by a scalar concept. There are however more complex, multidimensional sound features such as sound colour that probably has to be represented by many psychometric functions.

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