



FINITE-AMPLITUDE STANDING WAVES IN CYLINDRICAL AND SPHERICAL ACOUSTIC RESONATORS

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ABSTRACT

The paper deals with the problems of finite-amplitude standing waves in acoustical resonators of cylindrical and spherical shape filled with a viscous and thermal conducting gas. Frequency equations are derived in the first approximation to estimate the resonance frequencies and to show that the higher modes of the resonators are not integer multiples of the mode fundamental, similarly as in the case of resonators with variable cross-section. That is why the nonlinear effects are partially suppressed and why it is possible to generate high-amplitude acoustic fields. Further it is derived the system of two coupled one-dimensional nonlinear partial differential equations in conservative form from the fundamental equations of gasdynamics to describe finite-amplitude acoustic field in the gas-filled resonators. High resolution central difference scheme is used for numerical integration of the model equations to study properties and behaviour of standing acoustic waves of extreme amplitudes.

1. INTRODUCTION

Purpose of this article is formulation of model equations describing finite-amplitude acoustic standing waves in cylindrical and spherical gas-filled acoustic resonators using a set of nonlinear convection-diffusion equations. The reason for this is a fact, that a general analytical solution for equations of these types has not been yet found and thus, numerical approach is necessary for study of finite-amplitude acoustic waves behaviour. In literature, there are a lot of approximate difference schemes presented for nonlinear conservation laws and for closely related convection-diffusion equations numerical integration.

In one-dimensional form, the convection-diffusion equation in the conservative form can be written as

$$\frac{\partial}{\partial t} \mathbf{u}(x,t) + \frac{\partial}{\partial x} \mathbf{f}[\mathbf{u}(x,t)] = \frac{\partial}{\partial x} \mathbf{Q} \left[\mathbf{u}(x,t), \frac{\partial}{\partial x} \mathbf{u}(x,t) \right], \quad (1)$$

where \mathbf{u} is an N -vector of conserved quantities, $\mathbf{f}(\mathbf{u})$ is a nonlinear convection flux and $\mathbf{Q}(\mathbf{u}, \mathbf{u}_x)$ is a dissipation flux.

2. MODEL EQUATION

Acoustic standing waves of the finite amplitude in air-filled resonator can be described by using the fundamental equations of gas dynamics, see for example [1], [2], [5]. In conservative form, set of two one-dimensional equations in the second approximation is derived

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho v) &= -\frac{n}{r} \rho v, \\ \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial r} \left[\rho_0 v^2 + \frac{c_0^2}{2\rho_0}(\gamma-1)\rho^2 - c_0^2(\gamma-2)\rho \right] &= -\frac{n}{r} \rho_0 v^2 + b \frac{\partial}{\partial r} \left(\frac{\partial v}{\partial r} + \frac{n}{r} v \right), \end{aligned} \quad (2)$$

where v is acoustic velocity vector component along the radial coordinate, ρ is density, ρ_0 is its ambient value, r is spatial coordinate along the resonator body, t is time, c_0 is small-signal speed of sound, $\gamma = c_p / c_v$ is ratio of constant pressure and constant volume specific heats and $b = \zeta + 4\eta/3 + \kappa(1/c_v - 1/c_p)$ is the diffusivity of sound. Here η and ζ are coefficients of shear and bulk viscosity. These equations describe finite-amplitude acoustic waves in constant cross-section resonator ($n=0$), cylindrically-shaped resonator ($n=1$) and spherical resonator ($n=2$). Boundary conditions for equations set (2) have form

$$\frac{\partial \rho}{\partial r} = \frac{\partial p}{\partial r} = v = 0 \quad \text{for } r = r_1 \quad \text{and} \quad \frac{\partial \rho}{\partial r} = \frac{\partial p}{\partial r} = v = v_m \quad \text{for } r = r_2,$$

where v_m is velocity of vibration of external (right in the case of the constant cross-section resonator) boundary of the resonator. In the case of cylindrical and spherical resonator, $r_1=0$ means that the resonators have no internal boundary.

For purposes of numerical integration, new dimensionless variables are useful to be introduced

$$V = \frac{v}{\pi c_0}, \quad \Lambda = \frac{\rho}{\rho_0}, \quad M = \Lambda V, \quad X = \frac{r-r_1}{r_2-r_1}, \quad T = \omega t,$$

model equations set (2) then transforms to form

$$\begin{aligned} \frac{\partial \Lambda}{\partial T} + \frac{\partial}{\partial X} \left(\frac{M}{\Omega} \right) &= -\frac{nM}{\Omega X}, \\ \frac{\partial M}{\partial T} + \frac{1}{\Omega} \frac{\partial}{\partial X} \left[\left(\frac{M}{\Lambda} \right)^2 + \frac{\gamma-1}{2\pi^2} \Lambda^2 - \frac{\gamma-2}{\pi^2} \Lambda \right] &= -\frac{n}{\Omega X} \left(\frac{M}{\Lambda} \right)^2 + \frac{G_{TV}}{\pi^2 \Omega^2} \frac{\partial}{\partial X} \left[\frac{\partial}{\partial X} \left(\frac{M}{\Lambda} \right) + \frac{n}{X} \frac{M}{\Lambda} \right]. \end{aligned} \quad (3)$$

where

$$\Omega = \frac{\omega}{\omega_0}, \quad G_{TV} = \frac{\omega b}{\rho_0 c_0^2}$$

and $\omega_0 = \pi c_0 / l$ is the first fundamental angular frequency for planar waves in constant cross-section resonator, $l = r_2 - r_1$. The equations set (3) can be solved using a high-resolution numerical scheme for convection-diffusion equations in conservative form, see for example [3], [4]. Acoustic pressure distribution along the resonator body can be obtained from solution of equations (3) using the state equation for an ideal gas, see [5],

$$p = p_0 + \rho_0 c_0^2 \left[\Lambda - 1 + \frac{\gamma-1}{2} (\Lambda - 1)^2 - \frac{G_T}{\Omega X^n} \frac{\partial}{\partial X} \left(\frac{X^n M}{\Lambda} \right) \right], \quad (4)$$

where

$$G_T = \frac{\kappa\omega}{\rho_0 c_0^2} \left(\frac{1}{c_v} - \frac{1}{c_p} \right).$$

Equation (4) is written in the second approximation.

3. NUMERICAL RESULTS

A semi-discrete scheme of the second order, see [3], was implemented for numerical integration of set (3) to study extreme-amplitude planar, cylindrical and spherical standing acoustic waves. Results presented in this paper are computed for room-conditions air-filled resonators where $r_1=0$ cm, $r_2=10$ cm, $V_m=5 \times 10^{-4}$. Figures presented below show pressure waveform in different types of resonators. It is evident that in the case of cylindrical and spherical waves (figures 2 and 3) no shock-waves are formed and amplitudes of acoustic waves are stronger. It is because resonant conditions are not fulfilled for higher harmonic components of the fundamental mode.

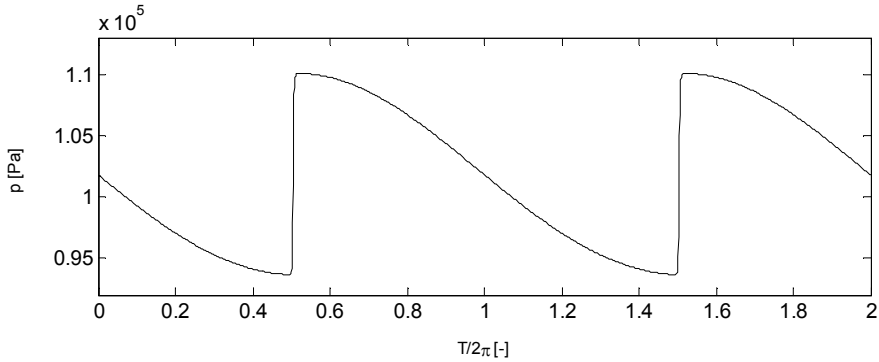


Figure 1: Pressure near one end of constant cross-section resonator with planar waves, $\Omega = 1$.

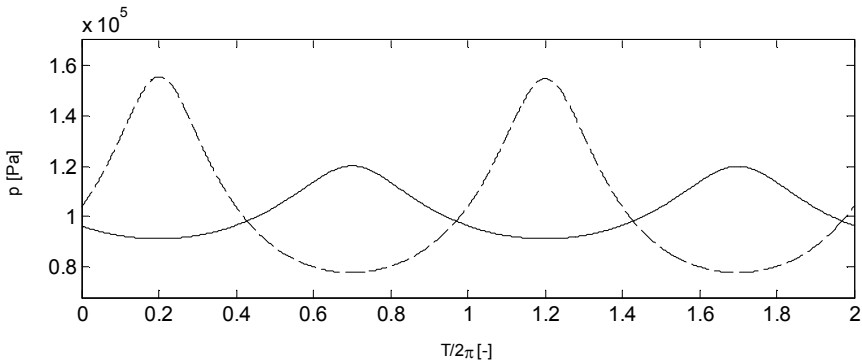


Figure 2: Pressure at the center (dashed line) and at the boundary (solid line) of cylindrical resonator, $\Omega = 1.235306$.

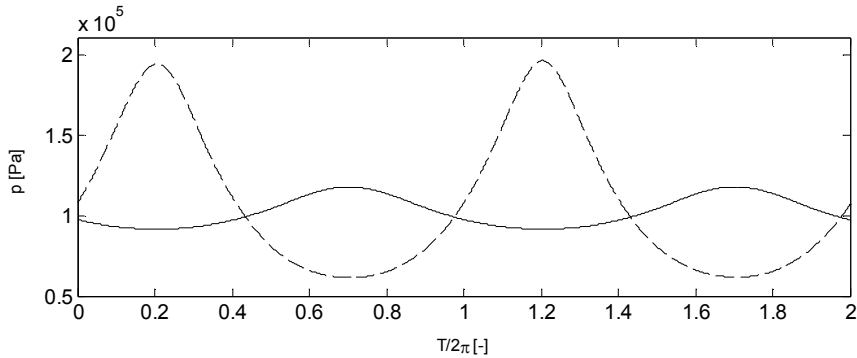


Figure 3: Pressure at the centre (dashed line) and at the boundary (solid line) of spherical resonator, $\Omega = 1.447033$.

4. CONCLUSIONS

Model equations for nonlinear standing waves in variable cross-section acoustic resonators filled with a thermoviscous fluid are possible to be formulated in convection-diffusion equations form and thus they can be numerically solved using well known schemes for integration of nonlinear conservation laws equations written in the conservative form. These schemes are also constructed for solution of multidimensional problems and so description of strong acoustic fields is straightforwardly possible to extend into more dimensions.

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