LI OTWARTE SEMINARIUM Z AKUSTYKI
Gdańsk – Sobieszewo, 6–10.09.2004

ACTIVE HARMONIC SUPPRESSION IN THE NONLINEAR ACOUSTICAL RESONATOR

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ABSTRACT

The method of the active second harmonic suppression in nonlinear acoustical resonator is analyzed in this work. The finite-amplitude standing waves in a resonator of a constant diameter can be described by means of the inhomogeneous Burgers equation. The resonator is driven by a piston whose motions are characterized by two superposed sinusoidal motions. The frequency of the first motion f is equal to the resonator eigenfrequency and the frequency of the second one is $2f$ and its the phase shift is 180 degrees. New approximate solution of the inhomogeneous Burgers equation for real fluid in stationary state regime is shown in this work. This solution is derived using the Prandtl's technique. Validity of the approximate solution is verified by comparison with the numerical one.

1. INTRODUCTION

Using of nonlinear standing waves is limited by the nonlinear attenuation that causes the acoustic saturation effects. The important characteristic of the resonator is the quality factor Q that shows how many times the amplitude of the steady-state wave is greater then the amplitude of vibration of the exciting piston. The resonators of the high Q-factor are used for thermo-viscous engines, acoustic compressors, chemical disintegrating devices. The Q-factor depends on the amplitude of the vibrating piston due to nonlinear attenuation. The nonlinear attenuation is connected with nonlinear acoustic wave interactions when we can observe generation of higher harmonics. As the thermo-viscous attenuation is proportional to the square of frequency it is possible to decrease the nonlinear attenuation by suppression of the wave cascade processes,

There is a number of methods for suppression of the nonlinear attenuation and thus for increasing of the quality factor of the given resonator. One of these methods is based on the active suppression of the second harmonic component of the sound wave. The other method is a passive one which uses selective frequency absorption [1]. It is also possible to increase the quality factor of the resonator using the macrosonic resonance synthesis, which is based on the utilization of the resonators of variable diameters.

This work is focused on looking for an approximate solution of the inhomogeneous Burgers model equation, which is used for description of an active harmonic suppression of higher harmonics inside a resonator. The excitation of this acoustic field is made in terms of a piston that vibrates at two frequencies and enables to control generation of higher harmonics, thus to increase the *Q*-factor.

2. MODEL EQUATION

Consistent with the second-order nonlinear theory, steady-state standing waves can be represented as a superposition of simple waves propagating in opposite directions. In the resonator we can imagine the sound field as a superposition of simple waves propagating in opposite directions, which are assumed to not interact in the volume of the resonator and they are coupled only by the conditions on the walls of resonator. We also neglect the fact that the driving piston is moving and thus the position of the boundary of the resonator is unvarying with the time. This assumption is acceptable for very small amplitude of driving piston. Then it is possible to describe the acoustic field in the resonator by means of the inhomogeneous Burgers equation (see [2])

$$\frac{\partial V}{\partial s} - V\frac{\partial V}{\partial y} - \frac{1}{\Gamma}\frac{\partial^2 V}{\partial y^2} = \sin(y) + p\sin(2y+\pi) \cdot \tag{1}$$

Eq. (1) will be solved in the stationary state by the Prandtl's technique [4], [5].
As Γ→∞, Eq. (1) reduces to

$$-V\frac{dV}{dy} = \sin(y) + p\sin(2y+\pi) \cdot \tag{2}$$

The exact solution of this equation it is called the outer solution V^o

$$V^o = \sqrt{2 + p + 2\cos(y) - p\cos(2y)} \tag{3}$$

For large Γ the solution of the reduced equation (2) is close to the exact solution of (1) except a small interval at the end point *y=0*, where the exact solution changes quickly in order to retrieve the boundary condition *y(0)=0.* This small interval, across which *y* changes very rapidly, represents the shock region. To determine the solution valid in this region, we magnify it using the stretching transformation

$$\zeta = \Gamma y \, .$$

With this transformation Eq. (1) becomes

$$-V\frac{dV}{d\zeta} - \frac{d^2V}{d\zeta^2} = \frac{1}{\Gamma}[\sin(y) + p\sin(2y+\pi)],$$

which for Γ→∞ reduces to

$$V\frac{dV}{d\zeta} + \frac{d^2V}{d\zeta^2} = 0 \cdot$$

We denote the general solution of this equation by V^i and call it the inner one

$$V^i = \sqrt{2A}\tanh\left[\sqrt{\frac{A}{2}}(\zeta + B)\right],$$

where A and B are constants. This solution is valid within the shock region. The constant *B* is equal to zero, because *V(0)=0.* To determine the constant *A* we will use the matching principle

$$\lim_{y\to 0} V^o(y) = \lim_{\zeta\to\infty} V^i(\zeta) = V^{io} \cdot$$

Hence *A=2.* An approximate solution to Eq. (1) is given by the outer solution V^o for y outside the shock region and by the inner solution V^i for y inside the shock region. To compute V as a function of all y we form a single uniformly valid solution called the composite solution V^c

$$V^c = V^o + V^i - V^{io} + O\left(\frac{1}{\Gamma}\right).$$

The composite solution is valid over the whole interval of y including the gap between the outer and inner regions. We have

$$V^c = \sqrt{2 + p + 2\cos(y) - p\cos(2y)} + 2\tanh[\Gamma y] - 2 + O\left(\frac{1}{\Gamma}\right). \tag{4}$$

The composite solution represents the approximate solution of the inhomogeneous Burgers equation (1) for real fluid and large Γ.

3. RESULTS

The inhomogeneous Burgers equation (1) was numerically solved in the frequency domain by means of the standard Runge-Kutta method of the fourth order. The first 200 harmonics were used. The comparison of the composite solution (4) and the numerical one for Γ=100 and p=10 is shown in Fig 1. We can observe that the solutions are in a very good agreement. This figure also demonstrates the contribution of the inner and outer solution to the composite one. The time evolution is depicted also in smaller time interval in Fig 2.

4. CONCLUSIONS

To solve inhomogeneous Burgers equation analytically we used the Prandtl's technique of matched asymptotic expansions. By means of this method we found the new approximate composite solution that was compared with the numerical one. This composite solution limits for Γ→∞ to the known solution for ideal fluid presented in [3]. The comparisons between numerical and composite solution demonstrate that the composite solution gives very good results for Γ > 50.

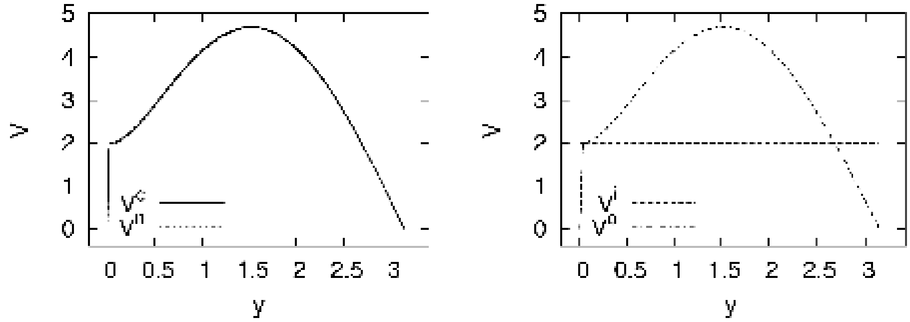

Figure 1: One half-period of time evolution of velocity for Γ=100, p=10. Comparison of the composite solution for real fluid V^c and numerical solution V^n (left). Time evolution of the inner solution V^i and the outer solution V^o (right).

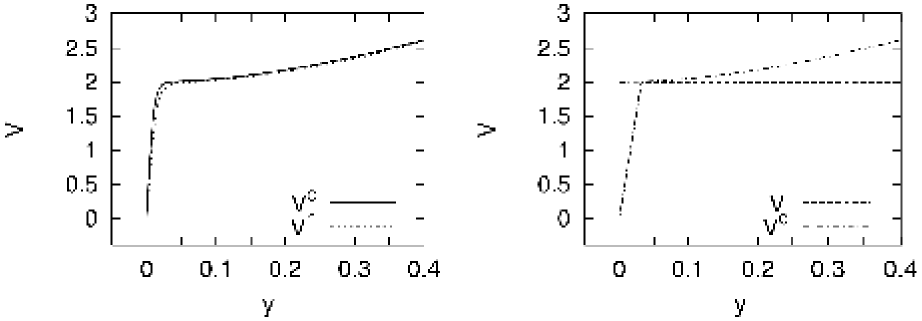

Figure 2: Shock region detail of time evolution of velocity for $\Gamma=100$, $p=10$. Comparison of the composite solution for real fluid V^c and numerical solution V^n (left). Time evolution of the inner solution V^i and the outer solution V^o (right).

ACKNOWLEDGEMENTS

This work was supported by the CTU research program 16:CEZ:J04/98:212300016.

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