

## Subject card

|                                             |                                                                                                                                                                   |                                                          |                                         |                                     |                                                |            |     |
|---------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|-----------------------------------------|-------------------------------------|------------------------------------------------|------------|-----|
| Subject name and code                       | Mathematical Analysis III, PG_00188841                                                                                                                            |                                                          |                                         |                                     |                                                |            |     |
| Field of study                              | Analiza matematyczna III                                                                                                                                          |                                                          |                                         |                                     |                                                |            |     |
| Date of commencement of studies             | October 2026                                                                                                                                                      |                                                          | Academic year of realisation of subject |                                     | 2027/2028                                      |            |     |
| Education level                             | Bachelor's studies                                                                                                                                                |                                                          | Subject group                           |                                     | Obligatory subject group in the field of study |            |     |
| Mode of study                               | full-time studies                                                                                                                                                 |                                                          | Mode of delivery                        |                                     | at the university                              |            |     |
| Year of study                               | 2                                                                                                                                                                 |                                                          | Language of instruction                 |                                     | Polish                                         |            |     |
| Semester of study                           | 3                                                                                                                                                                 |                                                          | ECTS credits                            |                                     | 11.0                                           |            |     |
| Learning profile                            | academic                                                                                                                                                          |                                                          | Assessment form                         |                                     | exam                                           |            |     |
| Conducting unit                             | Institute of Mathematics -> Faculty of Mathematics, Physics and Informatics -> Rector                                                                             |                                                          |                                         |                                     |                                                |            |     |
| Name and surname of lecturer (lecturers)    | Subject supervisor                                                                                                                                                |                                                          | dr hab. inż. Jacek Gulowski             |                                     |                                                |            |     |
|                                             | Teachers                                                                                                                                                          |                                                          |                                         |                                     |                                                |            |     |
| Lesson types                                | Lesson type                                                                                                                                                       | Lecture                                                  | Tutorial                                | Laboratory                          | Project                                        | Seminar    | SUM |
|                                             | Number of study hours                                                                                                                                             | 60.0                                                     | 60.0                                    | 0.0                                 | 0.0                                            | 0.0        | 120 |
|                                             | E-learning hours included: 0.0                                                                                                                                    |                                                          |                                         |                                     |                                                |            |     |
| Learning activity and number of study hours | Learning activity                                                                                                                                                 | Participation in didactic classes included in study plan |                                         | Participation in consultation hours |                                                | Self-study | SUM |
|                                             | Number of study hours                                                                                                                                             | 120                                                      |                                         | 5.0                                 |                                                | 150.0      | 275 |
| Subject objectives                          | The aim of the course is to familiarize students with the concepts, theorems and methods of differential and integral calculus of functions of several variables. |                                                          |                                         |                                     |                                                |            |     |

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| Learning outcomes               | Course outcome                                                                                                                                                                                                                                                                       | Subject outcome                                                                                                                                                                                                                                                                                                                                                        | Method of verification                                                                                                                             |
|                                 | [MATL3_U02] is able to correctly use the concepts of mathematical analysis and measure theory, is able to apply the theorems and methods of these fields and is able to interpret the obtained results                                                                               | The student is able to present correct mathematical reasoning, formulate definitions and theorems in an understandable manner, both orally and in writing, is able to use the methods of differential calculus of several variables, is able to reduce an integral of a function of several variables to an iterated integral, is able to calculate surface integrals. | [SU3] opracowanie tekstowe/ praca pisemna<br>[SU4] test/egzamin - ustny lub pisemny<br>[SU8] obserwacja samodzielnej lub zespołowej pracy studenta |
|                                 | [MATL3_W02] knows and understands at an advanced level selected concepts, methods and theorems of mathematical analysis and measure theory, as well as basic examples both illustrating specific concepts in this field and allowing to refute false hypotheses or invalid reasoning | The student knows and understands the basic concepts and theorems of differential and integral calculus of functions of several variables, as well as the methods used in other branches of mathematics, including linear algebra and topology.                                                                                                                        | [SW4] test/egzamin - ustny lub pisemny<br>[SW1] wypowiedź ustna/rozmowa/ dyskusja                                                                  |
|                                 | [MATL3_W09] knows and understands the role and importance of proof in mathematics, as well as the concept of the importance of assumptions                                                                                                                                           | The student understands the concept of a mathematical theorem, is able to divide the theorem into three parts: assumption, assertion and proof, understands the role of assumptions in proof.                                                                                                                                                                          | [SW4] test/egzamin - ustny lub pisemny<br>[SW1] wypowiedź ustna/rozmowa/ dyskusja                                                                  |
|                                 | [MATL3_K01] is ready to acquire knowledge in order to solve cognitive and practical problems related to the field of mathematics and to use the opinions and assistance of experts                                                                                                   | The student understands the need for further education.                                                                                                                                                                                                                                                                                                                | [SK1] wypowiedź ustna/rozmowa/ dyskusja<br>[SK8] obserwacja samodzielnej lub zespołowej pracy studenta                                             |
| Subject contents                | [MATL3_U08] can formulate definitions and theorems in an understandable manner, both orally and in writing, and present correct mathematical reasoning on the learned topics                                                                                                         | The student is able to logically present mathematical reasoning, formulate definitions and provide examples of applications.                                                                                                                                                                                                                                           | [SU1] wypowiedź ustna/rozmowa/ dyskusja<br>[SU4] test/egzamin - ustny lub pisemny                                                                  |
|                                 | 1. Derivatives of functions of several variables. Partial derivatives, directional derivative, derivative (differential) - connections between these concepts.                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                    |
|                                 | 2. Higher order derivatives, Schwartz's theorem on the commutativity of partial differentiation. Taylor's formula, local extrema.                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                    |
|                                 | 3. Mappings $\mathbb{R}^n$ in $\mathbb{R}^m$ , Jacobian, diffeomorphism, theorem on local diffeomorphism. Theorem on implicit functions. Conditional extrema.                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                    |
|                                 | 4. Jordan measure in $\mathbb{R}^k$ .<br>5. Riemann integral in $\mathbb{R}^k$ . Fubini's theorem, theorem on the change of variables in a multiple integral.<br>6. Line integral. 1- and 2-forms. Integral of 1- and 2-forms. Green and Stokes' theorems.                           |                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                    |
| Prerequisites and co-requisites | Knowledge of the issues of mathematical analysis of functions of one variable at the level of lectures on Mathematical Analysis I and II.                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                                                    |
| Assessment methods and criteria | Subject passing criteria                                                                                                                                                                                                                                                             | Passing threshold                                                                                                                                                                                                                                                                                                                                                      | Percentage of the final grade                                                                                                                      |
|                                 | tests                                                                                                                                                                                                                                                                                | 51.0%                                                                                                                                                                                                                                                                                                                                                                  | 50.0%                                                                                                                                              |
|                                 | observation of the student's attitude                                                                                                                                                                                                                                                | 51.0%                                                                                                                                                                                                                                                                                                                                                                  | 0.0%                                                                                                                                               |
|                                 | exam                                                                                                                                                                                                                                                                                 | 51.0%                                                                                                                                                                                                                                                                                                                                                                  | 50.0%                                                                                                                                              |

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|----------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Recommended reading                                            | Basic literature                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <p>1. A. Birkholc, Mathematical Analysis. Functions of several variables (in Polish). PWN Warszawa, 1995.</p> <p>2. G.M. Fichtenholz, Differential and Integral Calculus, Parts I-III (in Polish). PWN Warszawa 1978.</p> <p>3. W. Kryszicki, L. Włodarski, Mathematical Analysis in Problems, Parts I-II. (in Polish) PWN Warszawa 1986.</p> <p>4.M. Laczko, V. Sos, Real Analysis, Series, Functions of Several Variables, and Applications. Springer 2013.</p> <p>5. W. Rudin, Principles of Mathematical Analysis (in Polish). PWN Warszawa 1982.</p> |
|                                                                | Supplementary literature                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | none                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|                                                                | eResources addresses                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Example issues/<br>example questions/<br>tasks being completed | <p>(1) Give the definition of the derivative of the function <math>f: \mathbb{R}^k \rightarrow \mathbb{R}^n</math> at <math>a \in \mathbb{R}^k</math>. Write the derivative matrix using partial derivatives.</p> <p>(2) State and prove Schwarz's theorem on the symmetry of the second-order differential.</p> <p>(3) State the implicit function theorem. Define the concepts that appear in it.</p> <p>(4) State Fubini's theorem for continuous functions of two variables <math>f: \mathbb{R}^2 \rightarrow \mathbb{R}</math>.</p> <p>(5) State and prove Green's theorem.</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Work placement                                                 | Not applicable                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

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