University of Gdańsk

Subject card

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| Subject name and code | Functional Analysis and Applications, PG_00207082 | | | | | | |
| Field of study | Mathematical Modeling and Data Analysis | | | | | | |
| Date of commencement of studies | October 2026 | | Academic year of realisation of subject | 2026/2027 | | | |
| Education level | Master's studies | | Subject group | Obligatory subject group in the field of study<br>Subject group related to scientific research in the field of study | | | |
| Mode of study | full-time studies | | Mode of delivery | at the university | | | |
| Year of study | 1 | | Language of instruction | Polish | | | |
| Semester of study | 2 | | ECTS credits | 5.0 | | | |
| Learning profile | academic | | Assessment form | exam | | | |
| Conducting unit | Faculty of Mathematics, Physics and Informatics -> Rector | | | | | | |
| Name and surname of lecturer (lecturers) | Subject supervisor | | dr hab. Tomasz Człapiński, prof. UG | | | | |
| | Teachers | | | | | | |
| Lesson types | Lesson type | Lecture | Tutorial | Laboratory | Project | Seminar | SUM |
| | Number of study hours | 30.0 | 30.0 | 0.0 | 0.0 | 0.0 | 60 |
| | E-learning hours included: 0.0 | | | | | | |
| Learning activity and number of study hours | Learning activity | Participation in didactic classes included in study plan | | Participation in consultation hours | | Self-study | SUM |
| | Number of study hours | 60 | | 3.0 | | 62.0 | 125 |
| Subject objectives | The aim of the course is to present the basic concepts and theorems of Functional Analysis and their applications. | | | | | | |

## Learning outcomes

| Course outcome | Subject outcome | Method of verification |
|---|---|---|
| [MMiADM2_W01] knows and understands in depth the theory of selected areas of mathematics, the complex relationships between them and their development trends | The student knows and understands the definitions and theorems of functional analysis that are the subject of the lecture, along with selected proofs and examples. | [SW4] test/exam - oral or written |
| [MMiADM2_U01] is able to construct mathematical reasoning: prove theorems, formulate, test and refute hypotheses by constructing and selecting counterexamples, choosing, if necessary, methods and tools from various branches of mathematics and related sciences | The student is able to prove theorems learned during the functional analysis lectures and refute erroneous hypotheses or invalid reasoning. | [SU4] test/exam - oral or written |
| [MMiADM2_W02] knows and understands in depth the role and importance of the construction of mathematical reasoning | The student knows the concepts of functional analysis presented in the lecture, knows proof methods and examples illustrating specific concepts in this field. | [SW4] test/exam - oral or written |
| [MMiADM2_W03] knows and understands to a deep degree a selected field of mathematics or related sciences and is able to understand the formulations of issues in this field that are still at the research stage and knows the connections between issues in this field and other areas of mathematics | The student knows and understands the connections between functional analysis and other areas of mathematics, such as topology or linear algebra. | [SW4] test/exam - oral or written |

## Subject contents

1. Metric spaces. Cauchy sequence, completeness, compactness. Space of continuous functions C[a,b].
2. Method of contracting maps in complete metric space. Applications to the study of nonlinear equations: integral equations, Cauchy problem for first-order differential equations.
3. Normed spaces, Banach spaces. Examples of Banach spaces, l^p, L^p(a,b) spaces. Non-compactness of the ball in infinite-dimensional spaces.
4. Linear mapping in normed spaces and Banach spaces. Boundedness and continuity of linear operator, mapping norm. Kernel and image of a mapping, inverse mapping. Space of linear mappings. Closed graph theorem. Hahn-Banach theorem.
5. Linear functionals on normed space. The space conjugate to the normed space. The form of linear functionals on the spaces l^p, L^p(a,b).
6. Unitary spaces. Examples of unitary spaces. Schwarz's inequality. Parallelogram identity. Polarization formula for the dot product.
7. Hilbert spaces. Orthogonal projection theorem. Orthogonal decomposition theorem. Riesz representation theorem. Fourier series, Bessel's inequality, complete orthonormal system. Adjoint and self-adjoint operator.
8. Weak topologies in normed space.
9. Elements of operator algebra theory. Banach algebras and C*-algebras.
10. Eigenvalues and eigenvectors of a linear operator. Applications of functional analysis in quantum mechanics.

## Prerequisites and co-requisites

N/A

## Assessment methods and criteria

| Subject passing criteria | Passing threshold | Percentage of the final grade |
|---|---|---|
| tests | 51.0% | 45.0% |
| activity in classes | 51.0% | 5.0% |
| exam | 51.0% | 50.0% |
| observation of the student's attitude | 51.0% | 0.0% |

## Recommended reading

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| Basic literature | 1. J.C. Robinson, "An Introduction to Functional Analysis", Cambridge University Press 2020.<br>2. J. Musielak, "Wstęp do analizy funkcjonalnej", PWN 1989. |
| Supplementary literature | 1. W. Kołodziej, "Wybrane rozdziały analizy matematycznej", PWN 1982.<br>2. A. Alexiewicz, "Analiza funkcjonalna", PWN 1968. |
| eResources addresses | |

## Example issues/ example questions/ tasks being completed

not included

## Work placement

Not applicable